Path integral measure in Regge calculus from the functional Fourier transform

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Abstract

The problem of fixing measure in the path integral for the Regge-discretised gravity is considered from the viewpoint of it's "best approximation" to the already known formal continuum general relativity (GR) measure. A rigorous formulation may consist in comparing functional Fourier transforms of the measures, i.e. characteristic or generating functionals, and requiring these to coincide on some dense set in the functional space. The possibility for such set to exist is due to the Regge manifold being a particular case of general Riemannian one (Regge calculus is a minisuperspace theory). The two versions of the measure are obtained depending on what metric tensor, covariant or contravariant one, is taken as fundamental field variable. The closed expressions for the measure are obtained in the two simple cases of Regge manifold. These turn out to be quite reasonable one of them indicating that appropriately defined continuum limit of the Regge measure would reproduce the original continuum GR measure.

Regge calculus still remains the most natural discrete regularisation of general relativity (GR) promising from the viewpoint of constructing well-defined quantum gravity theory [1]. The result of quantisation being expressed in the form of the path integral, the key question is that of the choice of the integration measure. In particular, the earliest quantum formulation of 3D Regge calculus [2] is based on specific property of $6j$-symbols whose product for large values of arguments reduces to a kind of the path integral with the Regge calculus action, the arguments of $6j$-symbols being interpreted as linklengths. There are a number of models generalising these results to the physical 4D case [1], of which the Barrett-Crane one [3] attracts much attention for it analogously reproduces path integral with the Regge calculus action [4].

The path integral measure in numerical simulations is usually chosen as the simplest among the invariant ones [5]. Normalising the measure w.r.t. the DeWitt supermetric would allow to fix the measure uniquely [6]. However, in the 4D case this construction turns out to suffer from unrenormalisable UV divergences provided by singular nature of Regge manifold; discretisation of the Faddeev-Popov ghost field improves the situation, but the measure turns out to be singular at the point of superspace of metrics corresponding to the flat spacetime [7]. The reason for this is rather simple and connected with the change of gauge content of the theory in the flat spacetime when a certain variations of linklengths become gauge ones since they do not change geometry. The same unpleasant feature displays also in the canonical quantisation of the (3+1)D (continuous time) Regge calculus [8]. The singularity of the measure at the flat spacetime makes extracting physical consequences from the theory a difficult task because of the absence of the perturbative expansion around the flat spacetime. Therefore it may be useful to study the problem of quantum Regge calculus within another framework. Thus far Regge calculus has been treated as independent theory without any reference to the continuum GR. Now consider it as simply approximation to or regularisation of the already quantised continuum GR. So we need to define a notion of "the best approximation" to the known formal expression for the continuum measure

$$d\mu_{\rm C} = \prod_x (\det\|g_{ik}\|)^{-\frac{5}{2}} d^{10}g_{ik}. \tag{1}$$

(This simplest local invariant measure can be shown to correspond to the canonical quantisation of GR in a certain gauge, [9][1].) This notion can be given a strict sense by treating the measure as a functional $\int(\cdot)d\mu$ on the space of the functionals of metric. Since Regge metric is a particular case of general Riemannian one, the functional on Riemannian metrics can be viewed at the same time as that on Regge ones. Thus, the two measures, continuum (1) and discrete Regge one of interest $d\mu_{\rm R}$ can be defined on the same set of metric functionals. Looking for such set dense in the space of metric functionals in the appropriate topology and requiring that both measures would coincide

[1]Later the papers has been appeared, refs. [10, 11, 12] where the arguments in favour of the measure $d\mu_{\rm C} = \prod_x (\det\|g_{ik}\|)^{-3/2} g^{00} d^{10}g_{ik}$ instead of the eq.(1) used here were given. It was shown in that papers that the difference between the both expressions amounts to the new purely renormalisation terms in the perturbation theory for gravity without changing the structure of the theory.

on this set we can define $d\mu_{\mathrm{R}}$. The exponents of linear functionals of metric ("functional plane waves") just present such set, probably the only one which gives possibility to have expressions definable and calculable on the functional level.

The above approach is natural also from the axiomatic point of view where a functional subject to a certain set of axioms, the Osterwalder-Shrader ones, can be considered as functional Fourier transform of a measure of some quantum field theory [13], the so-called *characteristic functional* usually referred to in the physical literature as *generating functional*. The analog of the characteristic functional considered in our case takes the form

$$\hat{\mu}_{\mathrm{C}}(f) = \int e^{ig(f)} d\mu_{\mathrm{C}}(g) \tag{2}$$

where there are the two possibilities for the linear metric functional $g(f)$,

$$\int f_{ik} g^{ik} \sqrt{g} d^4x \equiv g_1(f) \tag{3}$$

or

$$\int f^{ik} g_{ik} \sqrt{g} d^4x \equiv g_2(f) \tag{4}$$

depending on what metric tensor, g^{ik} or g_{ik}, is chosen as true field variable; the $f_{ik}(x)$ or $f^{ik}(x)$ is probe function (since quantum fields are generally treated as distributions, the probe functions are usually supposed to be infinitely differentiable with compact supports). Strictly speaking, the measure involved in the definition of the characteristic functional should include also $\exp(-S)$, the S being the (Euclidean) gravity action. Occurence of this factor would make the explicit calculations not easier than defining and calculating the gravity path integral itself. Therefore we are trying to define Regge analog $d\mu_{\mathrm{R}}$ of the $d\mu_{\mathrm{C}}$ separately from $\exp(-S)$. A point of view on omitting this factor within strict framework of characteristic functional may consist in saying that the strong coupling limit ($S \rightarrow 0$) is considered.

Thus, our approach to definition of the Regge measure $d\mu_{\mathrm{R}}$ amounts to setting

$$\hat{\mu}_{\mathrm{R}}(f_{\mathrm{R}}) = \hat{\mu}_{\mathrm{C}}(f_{\mathrm{R}}) \tag{5}$$

on a discretised version f_{R} of the probe functions. The only natural choice for the tensor f_{R} on Regge manifold is to take it being piecewise-constant in the piecewise-affine frame, that is, constant on each the 4-simplex whenever g_{ik} is constant on it. Then one tries to define $\hat{\mu}_{\mathrm{C}}(f_{\mathrm{R}})$ (where f_{R} is not smooth but is a limit of smooth functions).

Strictly speaking, the measure $d\mu_{\mathrm{C}}$ does not exist as mathematical object, a regularisation is implicit. There should be some care with this regularisation. For example, the measure $d\mu_{\mathrm{C}}$ looks formally positive, and regularisation should keep this property. Convenience of the characteristic functional is, in particular, just that the positivity property looks rather simple if written in terms of this functional,

$$\sum_{\alpha,\beta=1}^{N} c_\alpha \bar{c}_\beta \hat{\mu}_{\mathrm{C}}(f_\alpha - \bar{f}_\beta) \geq 0 \tag{6}$$

for any sequence of the probe functions f_α and complex numbers c_α, $\alpha = 1, 2, ..., N$. If then $d\mu_{\mathrm{R}}$ is defined via (5), it's positivity immediately follows from (6). So we imply that positivity of the measure, if required, is ensured in the continuum GR; then it is guaranteed for our construction of the Regge measure too.

Now turn to our characteristic functional (2) which proves to be the product over points,

$$\hat{\mu}_{\mathrm{C}}(f) = \prod_x I(x), \tag{7}$$

of the factors (for $g(f) = g_1(f)$)

$$I = I_1 = \int e^{i f_{ik} g^{ik} \sqrt{g} d^4 x} (\det \| g_{ik} \|)^{-\frac{5}{2}+\epsilon} d^{10} g_{ik}. \tag{8}$$

Here a nonzero ϵ is introduced because, as mentioned above, the measure $d\mu_{\mathrm{C}}$ does not exist without regularisation, therefore not specifying the latter this measure can be understood whenever this is possible in the sense of analytical continuation from the sufficiently large positive ϵ where (8) can be defined to the point $\epsilon = 0$ of interest; the $d^4 x$ is an infinitesimal "bare" (i.e. corresponding to the Euclidean metric $g_{ik} = \delta_{ik}$) 4-volume associated to a point. To calculate this (and $I = I_2$ for $g(f) = g_2(f)$) note that

$$\begin{aligned} (\det \| g_{ik} \|)^{-\frac{5}{2}+\epsilon} d^{10} g_{ik} &= (\det \| g^{ik} \sqrt{g} \|)^{-\frac{5}{2}+\epsilon} d^{10} (g^{ik} \sqrt{g}) \\ &= (\det \| g_{ik} \sqrt{g} \|)^{-\frac{5}{2}+\frac{\epsilon}{3}} d^{10} (g_{ik} \sqrt{g}). \end{aligned} \tag{9}$$

Perform the following change of variables,

$$g^{ik} \sqrt{g} = \sum_A e^i_A \lambda_A e^k_A \tag{10}$$

where $e^i_A = 0$ at $A > i$, $e^i_A = 1$ at $A = i$ (this is the Gaussian decomposition of the symmetrical matrix into the product of a diagonal and twice a triangular one with unity diagonal elements). Integration over $d^6 e^i_A$ turns out to be Gaussian; the remaining integral over $d^4 \lambda_A$ is factorisable,

$$\begin{aligned} I_1 &= \prod_{A=1}^{4} (i\pi)^{2-\frac{A}{2}} (m_A)^{-\frac{1}{2}} \int \exp\left(i\lambda_A \frac{m_{A-1}}{m_A} \right) |\lambda_A|^{-\frac{1}{2}-\frac{A}{2}+\epsilon} d\lambda_A \\ &= 16 (i\pi)^3 (d^4 x \det f)^{-\epsilon} \prod_{A=1}^{4} \Gamma\left(\frac{1}{2} + \epsilon - \frac{A}{2} \right) \sin \frac{\pi}{2} \left(\frac{1}{2} - \epsilon + \frac{A}{2} \right) \\ &= 64 i \pi^5 (16 d^4 x \det f)^{-\epsilon} \frac{\Gamma(2\epsilon)}{\Gamma(1-2\epsilon)(1-2\epsilon)^2 (1-\epsilon)(3-2\epsilon)} \end{aligned} \tag{11}$$

where m_A is a diagonal minor of f_{ik} obtained by removing A first lines and A first rows from the matrix f_{ik} and taking the determinant. Expression for I_2 corresponding to the choice $g(f) = g_2(f)$ follows by replacing ϵ by $\epsilon/3$ and substituting f by f^{ik}.

Now consider reduction of $\exp(ig(f))$ and $\hat{\mu}_{\mathrm{C}}(f)$ to the Regge lattice; the functional relating these two reduced objects will be just the discrete measure of interest. The piecewise-affine frame is fixed by attributing the coordinates x^i_a, $i=1,2,3,4$ to each vertex a. The length squared $s_{(ab)}$ of the link (ab) connecting the vertices a and b,

$$s_{(ab)}=l^i_{ab}l^k_{ab}g_{ik}(\sigma),\quad l^i_{ab}\equiv x^i_a-x^i_b, \tag{12}$$

is a particular example of the so-called edge components $f_{(ab)}$ of a symmetrical second rank tensor f_{ik} constant in a simplex σ [14],

$$f_{(ab)}=l^i_{ab}l^k_{ab}f_{ik}(\sigma). \tag{13}$$

Here $g_{ik}(\sigma)$ and $f_{ik}(\sigma)$ are the values of the tensors in a simplex σ containing the link (ab). The edge components unambiguously parameterise a symmetrical rank two covariant tensor in a 4-simplex. But whereas $s_{(ab)}$ does not depend on the choice of σ containing the link (ab), the $f_{(ab)}$ may do so. However, the number of variables $f_{(ab)}$ Fourier-conjugate to the metric should be the same as the number of independent variables $s_{(ab)}$ parameterising the metric, i. e. the number of links. Therefore the condition is required that the variables $f_{(ab)}$ should not depend on $\sigma\supset(ab)$. The possibility to have $f_{(ab)}$ constrained by this condition becomes evident if one imagines that $f_{(ab)}$ are the new squared linklengths of our Regge manifold instead of $s_{(ab)}$, the scheme of linking and coordinates of vertices being the same (some of these linklengths can be made imaginary, if necessary, in the sense of analytical continuation); the metric tensor in the piecewise-affine frame in the 4-simplices of thus constructed Regge lattice will be just $f_{ik}(\sigma)$. Then according to the ref. [14] the functional (3) on Regge lattice functions takes the form

$$g_1(f_{\rm R})=2\sum_\sigma\sum_{(ab)\subset\sigma}f_{(ab)}\frac{\partial V_\sigma}{\partial s_{(ab)}}=2\sum_{(ab)}f_{(ab)}\frac{\partial V}{\partial s_{(ab)}}. \tag{14}$$

Here $V_{(\sigma)}$ is the volume of σ, $V=\sum_\sigma V_\sigma$ is the volume of the manifold (in the compact case).

Analogously, let $f^{(ab)}$ be independent variables living on the links. Let us define the contravariant symmetrical rank two tensor f^{ik} constant inside each the 4-simplex σ,

$$f^{ik}(\sigma)=\sum_{(ab)\subset\sigma}f^{(ab)}l^i_{ab}l^k_{ab}. \tag{15}$$

Using this anzats we get

$$g_2(f_{\rm R})=\sum_{(ab)}f^{(ab)}s_{(ab)}V_{(ab)}. \tag{16}$$

Here we have introduced notation for a volume associated to a link,

$$V_{(ab)}=\sum_{\sigma\supset(ab)}V_\sigma. \tag{17}$$

Next reduce the expression (7), (8) to the Regge lattice when $f(x)$ is piecewise-constant, $f(x)=f(\sigma)$ whenever $x\in\sigma$. Then $I(x)$ is piecewise-constant too, and for $g(f)=g_1(f)$ we have

$$\prod_x I(x)=\prod_\sigma\prod_{x\in\sigma}I_1(\sigma)=\prod_\sigma I_1(\sigma)^{N_\sigma}\sim\prod_\sigma(\det f(\sigma))^{-\epsilon N_\sigma} \tag{18}$$

(and analogously for $g(f) = g_2(f)$ with the replacement $\epsilon \to \epsilon/3$) where only dependence on f is shown. Here N_σ is a number of points contained in a simplex σ; of course, the continuum measure is defined in the limit $N_\sigma \to \infty$ starting from the originally finite N_σ. If integration over metric is made, information on the simplex size is lost, and the only choice symmetrical w.r.t. the different simplices and points is to consider N_σ being equal to the same value N for all the σ's. Then, if we keep N finite before taking the limit $\epsilon \to 0$, we can redefine $N\epsilon \to \epsilon$ (or $N\epsilon/3 \to \epsilon$ for $g(f) = g_2(f)$), so that

$$\hat{\mu}_{\rm C}(f_{\rm R}) \sim \prod_\sigma (\det f(\sigma))^{-\epsilon}. \tag{19}$$

This corresponds to the naive idea that the product over points should turn, up to a normalisation factor, into the same product but over simplices. Also we observe that it is namely the measure (1) for which this correspondence takes place; were the exponent there different from $-5/2$, as in the footnote following the eq. (1), the reduction to the Regge lattice like (18) could not be defined in such the simple way. Take into account parameterisation of the tensors f_{ik}, f^{ik} in terms of the edge components (13), (15). Then

$$\det\|f_{ik}(\sigma)\| = (\det\|l^i_{ab}\|)^{-2}\Delta_1(f;\sigma) \tag{20}$$

where $\Delta_1(f;\sigma)$ is the so-called bordered determinant [15] composed of the variables $f_{(ab)}$ living on the links (ab) belonging to σ. Note that $(\Delta_1(s;\sigma))^{1/2} = V_\sigma$, the volume of σ. The $\|l^i_{ab}\|$ means the matrix of any four link vectors l^i_{ab} of the simplex not laying in the same 3-plane. In the case $f = f^{ik}$ we can find even more simple expression,

$$\det\|f^{ik}(\sigma)\| = (\det\|l^i_{ab}\|)^2\Delta_2(f;\sigma), \quad \Delta_2 \equiv \sideset{}{'}\sum_{(a_ib_i)\subset\sigma} f^{(a_1b_1)}f^{(a_2b_2)}f^{(a_3b_3)}f^{(a_4b_4)} \tag{21}$$

where the summation runs over all the unordered combinations of the four links of the simplex, (a_ib_i), $i = 1, 2, 3, 4$ not laying in the same 3-plane.

Finally, we write out up to the normalisation factor the relation which fixes the Regge measure if we choose for the fundamental metric field the g^{ik},

$$\int \exp\left(2i\sum_{(ab)} f_{(ab)}\frac{\partial V}{\partial s_{(ab)}}\right) d\mu^{(1)}_{\rm R}(s) = \prod_\sigma (\Delta_1(f;\sigma))^{-\epsilon}, \tag{22}$$

or g_{ik},

$$\int \exp\left(i\sum_{(ab)} f^{(ab)}s_{(ab)}V_{(ab)}\right) d\mu^{(2)}_{\rm R}(s) = \prod_\sigma (\Delta_2(f;\sigma))^{-\epsilon}. \tag{23}$$

Consider the simplest case of Regge manifold consisting of the two identical 4-simplices σ_1, σ_2 with mutually identified vertices. Then $f_{ik}(\sigma_1) = f_{ik}(\sigma_2)$ or $f^{ik}(\sigma_1) = f^{ik}(\sigma_2)$ if parameterised by $f_{(ab)}$ or $f^{(ab)}$ according to (13) or (15). Now using $f_{(ab)}$ (or $f^{(ab)}$) and f_{ik} (or f^{ik}) as Fourier conjugate variables is equally convenient, because the number of links (ab) coincides with the number 10 of the components of f_{ik} (or f^{ik})

taken in one of the simplices. So we do not need to parameterise tensors by the edge components and can write immediately

$$\int \exp\left(i\frac{2}{4!}f_{ik}g^{ik}\sqrt{g}\det\|l^i_{ab}\|\right)d\mu^{(1)}_{\rm R}(g) = (\det\|f_{ik}\|)^{-2\epsilon} \tag{24}$$

or

$$\int \exp\left(i\frac{2}{4!}f^{ik}g_{ik}\sqrt{g}\det\|l^i_{ab}\|\right)d\mu^{(2)}_{\rm R}(g) = (\det\|f^{ik}\|)^{-2\epsilon} \tag{25}$$

instead of (22), (23) (again, up to a normalisation factor). Here we have taken into account that the product in the RHS consists of the two identical factors, so the exponent is simply rescaled, $\epsilon \rightarrow 2\epsilon$. The inverse Fourier transform is then straightforward and gives

$$d\mu^{(1)}_{\rm R} = d\mu^{(2)}_{\rm R} = (\det\|g_{ik}\|)^{-5/2}d^{10}g_{ik} \tag{26}$$

up to normalisation, or, in terms of linklengths,

$$d\mu_{\rm R} = V^{-5}d^{10}s, \tag{27}$$

the V being the volume of the simplex, $V = (\Delta_1(s))^{1/2}$.

The above example deals with the strongly curved spacetime; next consider the simplest Regge minisuperspace model of the flat spacetime. Take the flat 4-parallelepiped with all it's diagonals emitted from one of it's vertices and compactified toroidally by imposing periodic boundary conditions (on the linklengths). This is the simplest, consisting of 24 4-simplices elementary cell of the periodic Regge lattice [16] specified here by the conditions of compactness and flatness. The flatness means that the linklengths of the body and hyperbody diagonals can be expressed in terms of the linklengths of the 4 parallelepiped edges and 6 face diagonals. Equivalently, the metric g_{ik} can be taken the same in all the 24 4-simplices. Since the number of components g_{ik} coincides with the number 10 of independent linklengths, as in the example above, we again may work not passing to the variables s. Further, if we study the measure on the Regge minisuperspace constrained by additional conditions on the linklengths or metric, we need the same number of the conditions also on the Fourier conjugate variables f. In our case metric g_{ik} being the same in all the 24 equivalent 4-simplices, the Fourier transform of the measure of interest depends on $f^{ik}(\sigma)$ through the sum $f^{ik}(\sigma_1) + f^{ik}(\sigma_2) + ... + f^{ik}(\sigma_{24})$. The most symmetrical way of setting the conditions on $f^{ik}(\sigma)$ is to equate these for all the 4-simplices, $f^{ik}(\sigma) \equiv f^{ik}$, and analogously for f_{ik}. Finally, in the RHS we have the product of the 24 identical factors, so the exponent ϵ is rescaled to 24ϵ,

$$\int \exp\left(if_{ik}g^{ik}\sqrt{g}\det\|l^i_{ab}\|\right)d\mu^{(1)}_{\rm R}(g) = (\det\|f_{ik}\|)^{-24\epsilon}, \tag{28}$$

$$\int \exp\left(if^{ik}g_{ik}\sqrt{g}\det\|l^i_{ab}\|\right)d\mu^{(2)}_{\rm R}(g) = (\det\|f^{ik}\|)^{-24\epsilon}. \tag{29}$$

The answer is notationally the same as in the above example, eq. (26).

Thus, in the two examples, those of strongly curved and flat Regge manifolds we have obtained the same expressions for the measure written in terms of metric. This means that the measure cannot crucially depend on the curvature. On the other hand, the example of the flat spacetime might be relevant to the continuum limit of the Regge calculus. Indeed, if one triangulates a fixed smooth manifold with the help of Regge manifolds and tends the maximal linklength a of these manifolds to zero making triangulation finer and finer, then the angle defects of these Regge manifolds tend to zero too as Ra^2, R being typical curvature of the smooth manifold. The result we have obtained is just the expression for the continuum measure, although for a specific case when the product over points runs over only one point.

Despite that the two versions of the Regge measure coincide in the above simple cases, these are generally different, and thus far there is no indication which one of them is preferable. In general case we can write out explicit expression for the measure as convolution of elementary measures like (27) for all the 4-simplices,

$$\begin{aligned} d\mu^{(1)}_{\mathrm{R}} &= \prod_{(ab)}\left[d\left(\frac{\partial V}{\partial s_{(ab)}}\right)\right]\int\left\{\prod_{\sigma}\Delta_2(h_{(\sigma)};\sigma)^{-5/2+\epsilon}\right.\\ &\left.\cdot\prod_{(ab)}\left[\delta\left(\sum_{\sigma\supset(ab)}h^{(ab)}_{(\sigma)}-\frac{\partial V}{s_{(ab)}}\right)\prod_{\sigma\supset(ab)}dh^{(ab)}_{(\sigma)}\right]\right\} \end{aligned} \tag{30}$$

or

$$\begin{aligned} d\mu^{(2)}_{\mathrm{R}} &= \prod_{(ab)}\left[d(s_{(ab)}V_{(ab)})\right]\int\left\{\prod_{\sigma}\Delta_1(\tilde{s}^{(\sigma)};\sigma)^{-5/2+\epsilon}\right.\\ &\left.\cdot\prod_{(ab)}\left[\delta\left(\sum_{\sigma\supset(ab)}\tilde{s}^{(\sigma)}_{(ab)}-s_{(ab)}V_{(ab)}\right)\prod_{\sigma\supset(ab)}d\tilde{s}^{(\sigma)}_{(ab)}\right]\right\} \end{aligned} \tag{31}$$

where $\tilde{s}^{(\sigma)}_{(ab)}$ and $h^{(ab)}_{(\sigma)}$ are dummy variables living on the pairs 4-simplex — edge. It is taken into account that $\Delta_1^{-\epsilon}$ and $\Delta_2^{-5/2+\epsilon}$ or $\Delta_1^{-5/2+\epsilon}$ and $\Delta_2^{-\epsilon}$ are mutually connected by Fourier transform, as it follows from the relation of Δ_1 and Δ_2 to determinants of the co- and contravariant metric.

Probably the crucial difference between the two versions would display in the 2D model. There an analog of the eq. (30) could not be derived directly by Fourier transform of the continuum measure because the functional plane waves as functionals of $g^{ik}\sqrt{\det\|g_{ik}\|}$ do not depend on the conformal degree of freedom of the metric and thus do not form a dense set. But even being derived via analytic continuation from the dimensionality $n\neq 2$, the $d\mu^{(1)}_{\mathrm{R}}$ given by eq. (30) (where now V is the total square) is degenerate for it does not depend on the differential of the global conformal degree of freedom, while the $d\mu^{(2)}_{\mathrm{R}}$ does so. This can serve as some argument in favour of the version $d\mu^{(2)}_{\mathrm{R}}$, although the absence of the 2D puzzle can not be the criterium for the 4D case.

The expressions (30), (31) remind those for Feynman diagrams in the usual quantum field theory. However, the role of propagators is played by the fourth order polynomials raised to the negative half-integer power and with nontrivial position of zeroes. This makes analytic evaluation of nontrivial such graph quite difficult. But prior to that the problem of regularising the original continuous measure should be considered. In the simple examples of the present paper the explicit form of this regularisation turns out to be unimportant for the formal expression for the resulting Regge measure in the limit $\epsilon \rightarrow 0$. In the general case it is unclear whether expressions (30), (31) remain finite at $\epsilon \rightarrow 0$ without any additional regularising $d\mu_{\rm C}$ or not; if not, this would mean that the final formal expressions for $d\mu_{\rm R}$ would generally depend on this regularisation.

This work was supported in part by the grants E00-33-148 and RFBR No. 00-15-96811.

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